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Double Stratification Effects on MHD Tangent Hyperbolic Nano-fluid Flow with Variable Viscosity and Heat Transfer

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Abstract

Present study offerings two-dimensional flow of magneto hydrodynamic tangent hyperbolic fluid along with double stratification effects, heat transfer and nano-particles to a widening sheet. The mathematical modeling of existing flow study provides non-linear partial differential equations which are then convert to ordinary differential equations via suitable scale transformations. The resulting equations are then resolved using the BVP4C (MATLAB package). The performance of physical parameters involved on temperature, concentration and velocity are construed in aspect. Furthermore, the calculated outcomes are linked with the prevailing works to authenticate the accurateness of the results, as a result current consequences have enough similarity to informed facts.

***Keywords**: Double Stratification, Hyperbolic Tangent nanofluid, flow of MHD, Variable Viscosity, Heat Generation, Extending Sheet, BVP4C.*

1. Introduction:

The steady flow on an extending sheet along with heat transfer is a traditional problematic in fluid mechanics. In earlier some years, flow of boundary layer on an extending sheet become more of reputation because it exists in different fields of mechanical and engineering progressions, metallurgical and chemical progressions like fibre glass manufacture, hot rolling, melt-spinning etc. Through the procedure of polymer sheets in businesses, an incessant swelling of polymer extracted from the mechanical roller perishes, then shrill polymeric sheets are designed by the incessant stretching of the bound polymer that hardened by sustaining or cooling by close interaction with an invasive liquid.

Hyperbolic tangent fluid is the conspicuous branch of non-Newtonian fluids. Friedman et al. [1] mostly used tangent hyperbolic fluid model in magneto-rheological fluid obstacle loops. Tangent hyperbolic fluid is frequently used for altered tests in laboratory. Ali [2] examined the heat transmission features of an incessant extending sheet. An ignoble fluid contains the nanoparticles delayed in cautious heat transmission simple fluid along length gauges of 1100 mm is termed nano-fluid. Nano-fluid develops the convective coefficient of heat transmission and thermal conductivity of an ignoble fluids. Propylene glycol, water and oil are deprived electrodes of heat. If thermal conductivity of the fluids increases, numerous methods have been engaged in report. In these methods, the accumulation of nano-sized substantial particles in liquid is one of them. Choi et al. [3] initiate that thermal conductivity of an ignoble fluid by the

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accumulation of the nanoparticles augments twice. Das [4] clinched that the thermal conductivity depends on temperature by insertion of nanoparticles. Buongiorno et al. [5] gauged that the nanofluid particles are very cool for innovative atomic power plants. Malik et al. [6] scrutinized convective flow and boundary layer of nano-fluid mixed over an extending plate. The numerical result of nanofluid inspected by Khan et al. [7] over a plane extending sheet. Magneto hydrodynamic flow of nano-fluid premediated by Akbar et al. [8] over a perpendicular extending sheet along variable viscosity and considered the resistance possessions. The boundary layer and heat transfer flow of nano fluid particles scrutinized by Noghrehabadi et al. [9] through omission possessions. There are only a few researchers that worked on the models of non-Newtonian fluid [10-17]. Akbar et al. [18] scrutinized the breakdown of magnetic field earlier over an extending sheet in an interruption of gyrotic microbes and nano-fluid particles. Akbar et al. [19] considered Double-vague ordinary convective magneto hydro-dynamic boundary layer of nano fluid flow over an extending sheet.

The fluid properties change due to viscosity which depends on temperature of fluid. For liquids, the viscosity decreases due to increase in temperature while for gases and lubricating fluids, internal friction generates as increase in temperature, it will no lengthier stay constant. Due to inadequacy several researchers are delicate to investigate this effect on different variable viscosity models. Mukhpodhyay et al. [20] have demonstrated magneto hydrodynamic stream of viscid fluid along variable viscosity towards a stretchable sheet. The peculiarities of fluid diverge due to temperature dependent viscosity on viscoelastic fluids for e.g tangent hyperbolic fluid. For liquids, the viscosity decreases due to rise in temperature. In the case of gasses, there is enhancement in viscosity due to increase in temperature albeit for liquid viscosity and temperatures are inversely proportional to each other. To analyze the effect of changeable viscosity Reynolds consistency show is utilized by Massoudi and Phuoc [21]. The influence of changeable viscosity on peristaltic motion of third layer fluid was discussed by Elshehawey and Gharsseldien [22]. Martin [23] examined the analytic solution of visco-metric fluid flow using viscosity which depend on temperature. Khan et al [24] discoursed the changeable viscosity on parasitic flow of Jeffery fluid about awry divert in porous medium.

Double stratification arises due to fluctuations in temperature and concentration or by combination of liquids having different densities. This singularity has copious submissions in industry and engineering like diminishing and strengthening of copper wires. Rendering to Ibrahim and Makinde [25] scrutiny of varied convection in particularly stratified medium is an energetic problem because of its manifestation in geophysical tides. Mukhpodhyay et al. [26] analysed the possessions of thermal stratification on tide and heat transmission past a permeable vertical widening surface by means of lie group conversion method. The stimulus of double stratification develops imperious when conveyance of heat and mass arise instantaneously in convective tides. Rehman et al. [27] designated dual stratifcation in chemically sensitive casson fuid flow with mixed convection. [28] deliberate the variation of mixed convection in stratified flow of Eyring-Powell fluid executed by stretchable tending cylinder with heat generation/captivation. Arif et al. [38] explore the physical and computational aspects of normally applied magnetic field on non-Newtonian Prandtl-Eyring fluid flow over a starching sheet. Tasawar Hayat et al.[44] investigated the combined effects of Newtonian heating and internal heat generation/absorption in the two-dimensional flow of Eyring-Powell fluid over a stretching surface.

In the glimpse of aforementioned literature survey, it is contemplated that no venture has been done to scrutinize stimulus of variable thermal conductivity and magnetic field for flow of hyperbolic tangent non-Newtonian fluid. Therefore, the present endeavor is concentrated in this orientation. The intrinsic intention of this work is to scrutinize the influence of MHD and variable thermal conductivity of hyperbolic tangent fluid towards a deformable sheet. In previous literature tangent hyperbolic fluid along with nano particles are considered. But in this article tangent hyperbolic nano fluid along with double stratification effects are included. Moreover, viscosity of the fluid is assumed to be variable. The configuration of the present article is evolved in such a way that PDEs can be renovated into ODEs and then solved by Bvp4c numerically. The behavior of governing various non-dimensional parameters like power law index, Hartmann number, Weissenberg number, variable viscosity and Prandtl number has been examined for temperature and velocity profiles. The retrieved outcomes are delineated through tables and graphs in detail.

Nomenclature:

B_0	Magnetic field	η	Similarity independent variable
μ	Viscosity	v	Kinematics viscosity
U_w	Velocity of the stretching sheet	$U\infty$	Free-stream velocity
M	Hartman number	n	Power law index
α	Wall thickness parameter	C_f	Skin friction coefficient
C_p	Specific heat	σ	Electrical conductivity
x,y	Cartesian coordinates	Pr	Prandtl number
Ρ	Density	T	Temperature
uv	Velocity Components	Tw	Wall temperature
τw	Wall shear stress	$T\infty$	Temperature at infinity
θ	Dimensionless temperature	qw	Surface heat flux
Re	Reynolds number	Nu_s	Nusselt number
ε_1	Thermal stratification parameter	ε_2	Solutal stratification parameter
δ	Heat generation parameter	b,c,d,e	Constants
We	Weissenberg number	ϕ	Nano-particle concentration
D_B	Brownian motion coefficient	D_T	Thermophoresis coefficient
Φw	Surface Concentration	$\phi\infty$	Ambient concentration

Subscripts:

∞	Ambient condition
w	Condition at the wall

Suppose two-dimensional, steady, incompressible and tangent hyperbolic viscous flow with nano-fluid towards an extending sheet along the surface at y = 0 and flow lies in the region y > 0.

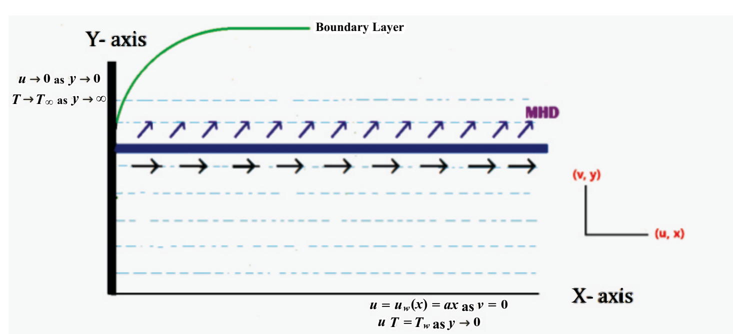

Figure 1: Geometry of the problem

We take the effect of magnetic field having strength B_0^2 that ordinary useful to the flow. The tensor of hyperbolic tangent of non-Newtonian fluid is defined as,

$$\bar{\tau} = [\mu_\infty + (\mu_0 + \mu_\infty)\tanh(\Gamma\bar{\dot{\gamma}})^n]A_1 \quad (1)$$

Where $\mu\infty$ shows the rate of infinite shear viscosity, n represent the law of power index, μ_0 shows the rate of zero-shear viscosity, Γ is material constant that depend on time and $\bar{\gamma}$ is given by

$$\bar{\dot{\gamma}} = \sqrt{\frac{1}{2}\sum_i\sum_j\bar{\dot{\gamma}}_{ij}\bar{\dot{\gamma}}_{ji}} = \sqrt{\frac{1}{2}\Pi} \quad (2)$$

Where $\Pi = \frac{1}{2}\,\mathrm{tr}(\mathrm{grad}\,V + (\mathrm{grad}\,V)^T)^2$

For simplicity we take $\mu\infty = 0$ and since tangent hyperbolic fluid model has shear thinning behaviour $\Gamma\ \bar{\gamma}<1$, so from equation (1)

$$\bar{\tau} = \mu_0[(\Gamma\bar{\gamma})^n]A_1 = \mu_0[(1+\Gamma\bar{\gamma}-1)^n]A_1$$

Using the binomial expansion, we have

$$\bar{\tau} = \mu_0[1+(\Gamma\bar{\gamma}-1)^n]A_1 \quad (3)$$

After solving the above equations by using boundary-layer estimation, the resulting equations of momentum, temperature and concentration are

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0, \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = (1-n)\frac{\partial^2}{\partial y^2}(vu) + \sqrt{2}vn\Gamma\left(\frac{\partial u}{\partial y}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2}{\rho}u, \quad (5)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left(D_B\left(\frac{\partial \phi}{\partial y}.\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right) + \frac{Q_0}{\rho c_p}(T - T_\infty), \quad (6)$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = D_B\frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right). \quad (7)$$

In above equations the component of velocity u is along x and v is the component of velocity along y-axis, ρ is a density of fluid, v represents kinematic viscosity, σ denote conductivity, Qu represent the heat generation coefficient, C_p represent precise heat at constant pressure, T denotes fluid temperature and ϕ symbolizes nanoparticle concentration. The boundary conditions are

$$u = u_w(x) = ax,\ v = 0,\ T = T_w,\ \phi = \phi_w, \text{ at } y = 0,$$
$$u \to 0,\ T = T_\infty,\ \phi \to \phi_\infty, \text{ at } y \to \infty. \quad (8)$$

In the above equations, u and v are components of velocity in x and y-axis individually, α shows thermal diffusivity, Γ is time, D_B is coefficient of Brownian motion and D_T is coefficient of thermophoresis transmission. Here $T_w = T_0 + bx$ denotes surface temperature, and $T\infty = T_0 + cx$ is flexible ambient temperature, T_0 is orientation temperature and b and c are positive constants. $\phi_w = \phi_0 + dx$ is the surface concentration while $\phi\infty = \phi_0 + ex$ is flexible ambient concentration, where ϕ_0 is orientation concentration, d and e are positive constants.

We use following transformations to convert the exhibited equations to the non-dimensional form (see Ref. [29, 37]),

$$\eta = \sqrt{\frac{a}{v}}y,\ u = axf'(\eta),\ v = -\sqrt{av}f(\eta)$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\ \beta(\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty}. \quad (9)$$

Now, by using equation (9), the Eqs. (4 - 8) becomes in the following form

$$f'^2 - ff'' - (1-n).f''' + (1-n)\theta_r\theta' f'' - nWef''f''' + n\sqrt{2}We\theta_r\theta' f''^2 + M^2 f' = 0 \quad (10)$$

$$\frac{\theta''}{Pr} + f\theta' + Nb\beta'\theta' + Nt\theta'^2 - \varepsilon_1 f' + \delta\theta = 0, \quad (11)$$

$$\beta'' + \frac{Nt}{Nb}\theta'' + Pr\,Lef\beta' - \varepsilon_2 f' = 0. \quad (12)$$

The corresponding boundary conditions are as follows

$$f = 0,\ f' = 1,\ \theta = 1 - \varepsilon_1,\ \beta = 1 - \varepsilon_2 \text{ at } \eta = 0,$$
$$f' \to 0,\ \theta \to 0,\ \beta \to 0, \text{ at } \eta \to \infty. \quad (13)$$

In above equations $M^2 = \frac{\sigma B^2}{a\rho}$ is Hartmann number, $We = \frac{\sqrt{2}a^{\frac{3}{2}}x\Gamma}{\sqrt{v}}$ is Weissenberg number, $P_r = \frac{v}{\alpha}$ is a Prandtl number, $\varepsilon_1 = \frac{c}{b}$ is thermal stratification parameter, $\varepsilon_2 = \frac{e}{d}$ is solutal stratification parameter and $\delta = \frac{Q_0}{a\rho C_p}$ is heat generation parameter.

The coefficient of skin friction is as follows

$$C_f = \frac{\tau_w}{\rho u_w^2}, \quad (14)$$

after simplifying this term, we have

$$\sqrt{R_e x}\,C_f = (1-n)f''(0) + \frac{n}{2}W_e\{f''(0)\}^2. \quad (15)$$

The local Nusselt number is as follows

$$Nu_x = \frac{xq_w}{k(T_w+T_\infty)}, \quad (16)$$

Here $qw = -k\left(\frac{\partial u}{\partial y}\right)_{y=y_0}$ the resulting equation is

$$(R_e x)^{-\frac{1}{2}} Nu_x = -\theta'(0). \quad (17)$$

The local "Sherwood number" is as under

$$Sh_x = \frac{xq_m}{D_B(\phi_w-\phi_\infty)}, \quad (18)$$

here $qm = -D_B\left(\frac{\partial \phi}{\partial y}\right)_{y=y_0}$, the resulting equation is

$$(R_e x)^{-\frac{1}{2}} Sh_x = -\beta'(0). \quad (19)$$

Table 1

The valuation of the coefficient of local skin resistance by fluctuating the Hartmann number.

M	Akbar et al. [38]	Malik et al. [30]	Hussain et al. [39]	Presented Result
0.0	-1	-1	-1	-1
0.5	-1.11803	-1.11802	-11180	-1.11805
1.0	-1.41421	-1.41419	-1.4137	-1.4140
5.0	-2.449449	-2.44945	-2.4495	-2.4497
10	-3.31663	-3.31657	-3.3166	-3.3169
100	-10.0498	-10.04981	-10.0500	-10.0499
500	-22.38303	-22.38294	-22.3835	-22.3837
1000	-31.63859	-31.63851	-31.6391	-31.6395

2. Numerical Solution:

Since the equations (10 12) associative with boundary conditions (13) are highly non-linear ODE's. Firstly, we covert it into first order ODE's. Then to find the solution of these ODE's, we use MATLAB package (BVP4C) method.

$$\left.\begin{aligned}
f &= y(1),\\
f' &= y(2),\\
y'(2) &= y(3),\\
y'(3) &= \frac{y^2(2)-y(1).y(3)+(1-n)\theta_r y(3).y(5)+n\theta_r Wey^2(3).y(5)+M^2 y(2)}{1-n+nWey(3)},\\
\theta &= y(4),\\
\theta' &= y(5),\\
y'(5) &= -P_r[\,y(1).y(5)+Nby(5).y(7)+Nty^2(5)+\varepsilon_1 y(2)+\delta y(4)],\\
\beta &= y(6),\\
\beta' &= y(7),\\
y'(7) &= \frac{NtP_r}{Nb}[y(1)y(5)+Nby(5)y(7)+Nty^2(5)+\varepsilon_2 y(2)+\delta y(4)]\\
&\quad +Ley(1)y(7)-\varepsilon_2 y(2).
\end{aligned}\right\} \quad (20)$$

BVP4C is a MATLAB package and it is an effective solver of boundary value problems. It solves the problems by using finite difference method. It flinches the solution with initial guess provided at an opening mesh points and deviations step size to getting the specified precision.

3. Results and Conversation:

In the recent flow conformation, the magneto hydro-dynamic boundary layer flow of the hyperbolic tangent nano-fluid on the elastic sheet is delimited. The numerical result of the modeled equations is originating with BVP4C (MATLAB package) technique. The accurateness of the calculated result is specialized by relating it with the prevailing literature (Malik et al. [30], Akbar et al. [38] and Hussain et al. [39]) through Table 1. We can observe this current solution is consistent with earlier results up to momentous number of characters. Now, let's deliberate graphical results of physical problems. Figure 1 demonstrates the possessions of the Hartmann number M on velocity outline by taking another parameters static. As the Hartmann number M relates to yield the Lorentz force, then for huge Hartmann numerical values M, Lorentz force is strengthened and offers enormous confrontation to the movement and therefore the velocity outline and heat transport rate reduces. Figure 2 trace the impacts of power law index on the velocity outline . It is observed that power rule index moderates the velocity outline . It is obvious that when power rule index rises, the type of shear thinning fluid transformed into shear thickness. Figure 3 appearances the stimulus of Weissenberg number on velocity outline . It is noted that the velocity outline decrease due to augmentation in Weissenberg number because Weissenberg number narrates directly through relaxation time and hereafter more confrontation is accessible. Figure 4 shows that the velocity outline decreases when thermal stratification parameter increases. Effective convective potential decreases at higher thermal stratification between ambient temperature and surface of sheet. Figure 5 describes the influence of the parameter of thermophoresis Nt on temperature. We find that for big values of the parameter of thermophoresis Nt, the temperature outline reduces. Because when we increase the value of the parameter of thermophoresis Nt, the fluid is transformed from hot to cold area. Figure 6 shows that when we increase the Lewis number , then temperature outline decreses. Figure 7 develops the fluxes in temperature outline contrary to Prandtl number Pr. Since for huge value of Prandtl number Pr, thermal conductivity reduces. Figure 8 shows that when we increase thermal stratification parameter , the temperature profile decreases. It is due to decrease in temperature alteration between surface of sheet and ambient fluid. Figure 9 indicate the effect of heat generation parameter on temperature outline . When we increase the heat generation parameter , the temperature outline also increase. An intensification in heat generation parameter also intensifications thermal boundary layer chunkiness. Figure 10 recommends that the improvement in parameter of thermophoresis Nt main to rise in concentration profile. Figure 11 exposes the disparities in attentiveness circulation for fluctuating values of the parameter of Brownian motion Nb. Since the symmetrical movement of nano-fluid particles is due to increase in parameter of Brownian motion Nb, hence it ultimately censures the nano-fluid particle absorption. The concentration outline that affected by Lewis number Le is represented through Figure 13. We observe that the decreasing behaviour of concentration profile is due to increase values of Lewis number Le. Figure 12 indicates that concentration outline decreases when the solutal parameter increases. Figure 14 shows the behaviour of the coefficient of local skin frication for several standards of physical parameters , M and n. We observe from these figures that the local skin friction decreases due to increase power law index n. Also, other characteristics of the coefficient of local skin friction are noted against Hartmann number M and Weiss Enberg number . Figure 15 ornamented to analyse the possessions of diverse parameters on the wall heat fluidity. Obviously, we observe that the ratio of surface heat transfer growths when Prandtl number Pr increases but the rate is decreasing when the values of the parameters of bronian motion parameter Nb and thermophoresis number Nt increases. Figure 16 display the behaviour of different values of thermophoresis number Nt and Lewis number Le on surface mass fluidity. It is detected that rate of mass transfer rises monotonically when we take the large values of Lewis number Le, parameter of Brownian motion Nb and thermophoresis number Nt.

Graphs:

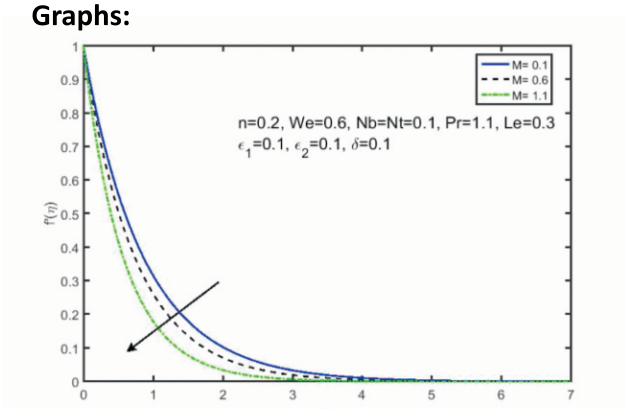

Figure 1. Effect of M $f'(\eta)$.

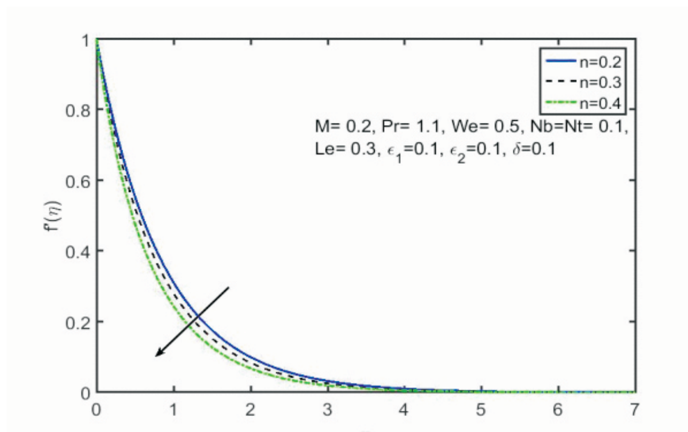

Figure 2. Effect of n $f'(\eta)$.

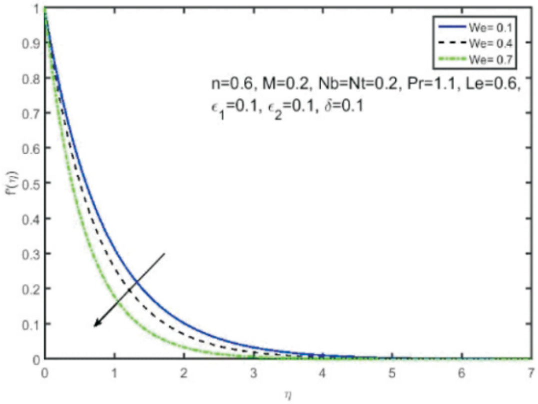

Figure 3. Effect of We c $f'(\eta)$.

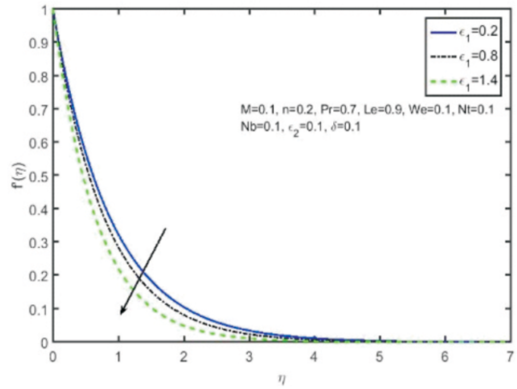

Figure 4. Effect of ε_1 c $f'(\eta)$.

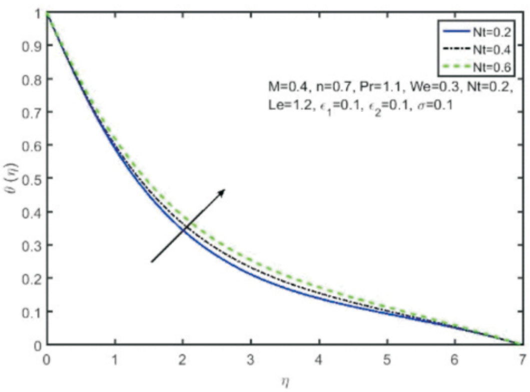

Figure 5. Effect of Nt on $\theta(\eta)$.

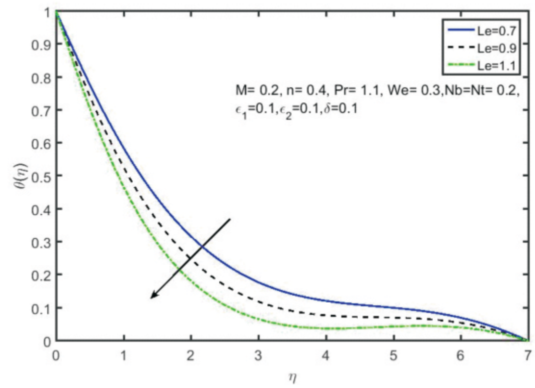

Figure 6: Effect of Le on $\theta(\eta)$.

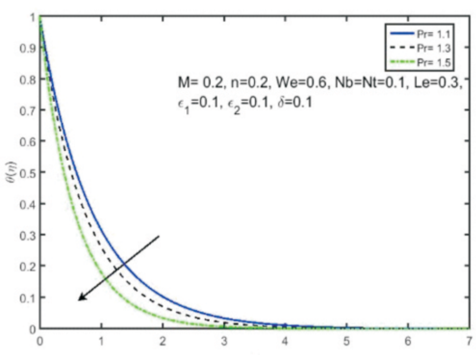

Figure 7: Effect of Pr on $\theta(\eta)$.

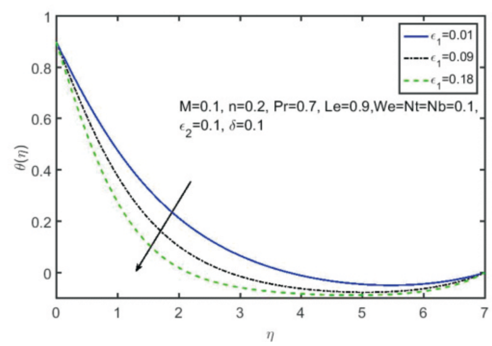

Figure 8: Effect of ε_1 on $\theta(\eta)$.

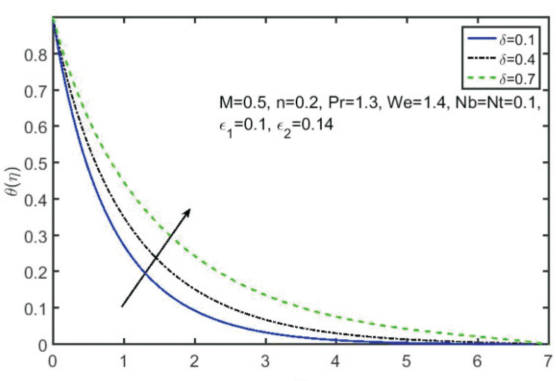

Figure 9: Effect of δ on $\theta(\eta)$.

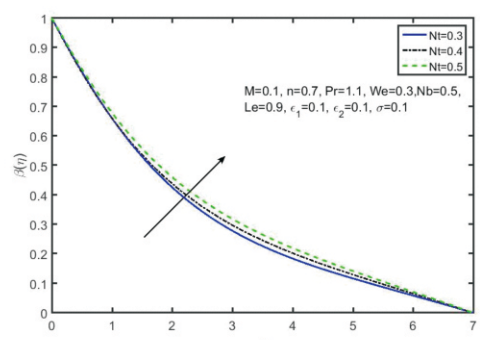

Figure 10: Effect of Nt on $\beta(\eta)$.

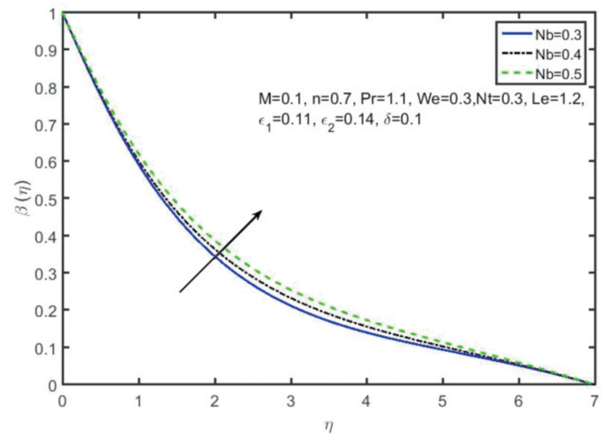

Figure 11: Effect of *Nb* on β(η).

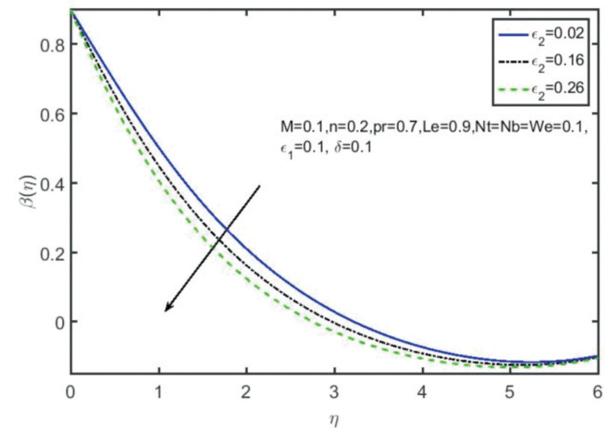

Figure 12: Effect of *Nb* on β(η).

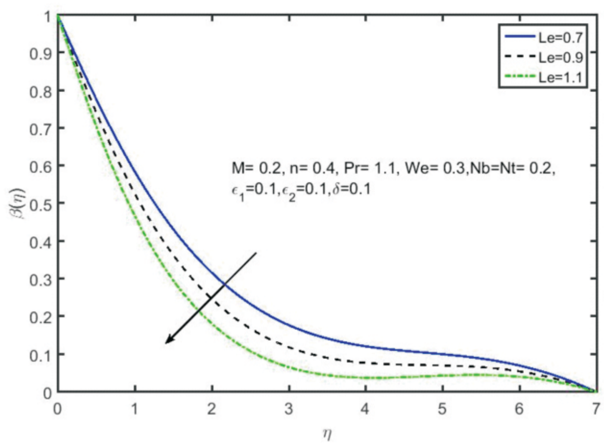

Figure 13: Effect of Le on β(η).

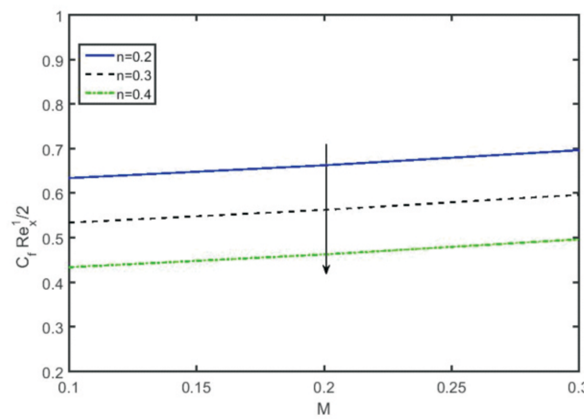

Figure 14. Influence of *n* and M on Local skin friction

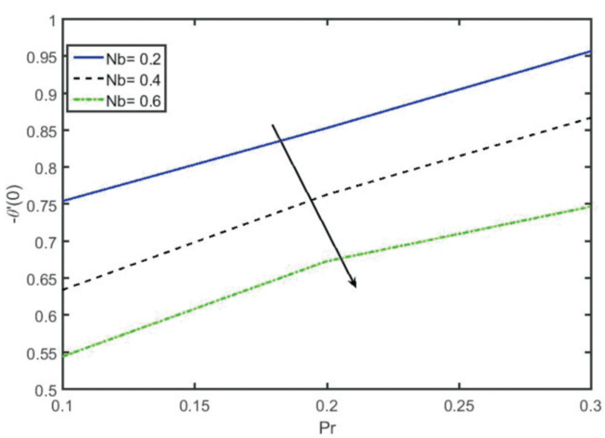

Figure 15: Effect of *Nb* and *Pr* c$\theta'(0)$.

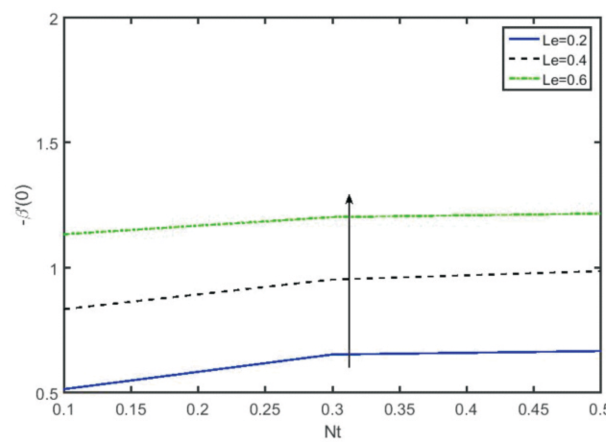

Figure 16: Effect of *Nt* and *Le* $-\beta'(0)$.

Conclusions:

The current study pronounces study of the hyperbolic tangent nano-fluid flow on a rectilinear elastic sheet underneath the influence of the magnetic ground. A model used for the tangent hyperbolic nano-fluid comprises the influences of the parameter of Brownian motion Nb and parameter of thermophoresis Nt. The result is not like the modeled limit, the question has been achieved that depends on Lewis number, Weisenberg number We, Hartmann number M, Prandtl number , power rule index n, parameter of thermophoresis number , Brownian movement , thermal stratification , solutal parameter and heat generation parameter . The main outcomes of this question are enumerated as follows:

- The velocity profile of the fluid is a declining function for all parameters (i.e.) power rule index n, Weiss Enberg number , Hartmann number M and thermal stratification parameter while for heat generation parameter , it is an increasing function.
- The temperature outline is a decreasing function for the parameter of Prandtl number and thermal stratification parameter while for the parameter of thermophoresis number , parameter of Brownian motion and heat generation parameter , it is an increasing function.
- The concentration outline is an increasing function for all parameters (i.e.) parameter of Brownian motion , thermophoresis number and thermal stratification parameter while for the parameters of Lewis number , solutal parameter and heat generation parameter , it is decreasing function.
- Hartmann number M increases the local skin resistance coefficient while it is decreasing function for power rule index and Weiss Enberg number .
- Prandtl number decreases the function of local Sherwood number while other parameters like Lewis number , thermophoresis and Brownian motion increases it.
- Local Nusselt number growths for Prandtl number while it falls down against the parameters of Lewis number , Brownian motion and thermophoresis .

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