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# Analytical Study of Some Chemical Engineering Equations Via an Efficient Method

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**Abstract**

*In this paper, another new strategy Aboodh transform homotopy perturbation technique (ATHPM) [4] has been utilized for comprehending the chemical engineering equations. This strategy is a coupling of Aboodh integral Transform and the homotopy perturbation technique. This strategy does not require discretization, and linearization. The nonlinear term in the differential equation can be effortlessly taken care of by homotopy perturbation technique and a few instances of these equations are unraveled as cases to show capacity and unwavering quality of the proposed strategy. The proposed technique gives the solution in an arrangement frame that merges quickly to the exact solution in the event that it exists. The comes about uncover that the blends of ATHPM strategies are very able, for all intents and purposes well fitting for taking care of such the problems. Additionally, this strategy is discovered that a superior elective strategy to some current strategies for such sensible* issues.

***Keywords**: Aboodh integral transform; Homotopy perturbation Method; He's polynomials, nonlinear system*

## 1. Introduction:

Differential equations arise in diverse fields to study natural problems: including mathematical physics applications, scientific and engineering categories, biological sciences, social science, and other fields of sciences etc. It is mostly difficult to get the exact or closed-form solutions of nonlinear problems.Differential equations are of basic significance on the grounds that a significant number of the physical wonders and procedures in application of real world problems. Current scientific advancements in mathematical physics and engineering issues have offered driving force to inquire about on strategies. Lamentably, such procedures which accept basically that a non-linear framework is relatively straight frequently have minimal physical justification. It has turned out to be crucial, to hypothesis as well as to the territories of down to earth application, that further advances be made. Lately the improvements of the fast computerized PC and expanded enthusiasm for non-linear systems have prompted an escalated investigation of solution of differential equations. Some indispensable integral transforms, for example, Laplace and Fourier and Sumudu transform strategies [1-8] are utilized to explain linear and nonlinear partial differential equations and utilize completion of these necessary transforms lies in their capacity to transform differential equations into algebraic equations

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which permits basic and efficient arrangement techniques. Be that as it may, utilizing vital transform in nonlinear differential equations may expand its unpredictability. As of late, numerous researchers have focused on discover the solutions of nonlinear differential equations by utilizing different techniques.

Distinctive specialists have connected numerous procedures among these are the Adomian decomposition technique [9], the homotopy perturbation strategies [10-11], the differential transform technique [12], the variational iteration technique [13-14], Exp-function method [15-16] and sine cosine strategy [17-18],etc. on scientific and physical models. Different ways have been proposed as of late to manage these nonlinearities, one of these blends of homotopy perturbation method and Aboodh transform which is ponders in this paper. The benefit of this strategy is its ability of consolidating two intense techniques for getting precise solution for nonlinear partial differential equations. Differential equations system is very significant for mathematicians, engineers, and physicists because in most of physical systems the input is not proportional to output in nature. The viability of the ATHPM in comprehending nonlinear chemical engineering equations systems is discussed in this work.

## 2. Aboodh Transform

For function *f(t),* the Aboodh transform is defined as

$$A\{f(t)\} = K(v) = \frac{1}{v}\int_0^\infty f(t)e^{-vt}\,dt. \qquad \textbf{(1)}$$

Some useful results are list below

$$A\{\frac{\partial f}{\partial t}(x,t)\} = vK(x,v) - \frac{f(x,0)}{v},$$

$$A\{\frac{\partial^2 f}{\partial t^2}(x,t)\} = v^2K(x,v) - \frac{1}{v}\frac{\partial f}{\partial t}(x,0) - f(x,0),$$

$$A\{\frac{\partial f}{\partial x}(x,t)\} = \frac{d}{dx}\{K(x,v)\},$$

$$A\{\frac{\partial^2 f}{\partial x^2}(x,t)\} = \frac{d^2}{dx^2}\{K(x,v)\}.$$

We can undoubtedly stretch out this outcome to the nth derivative by utilizing numerical enlistment.

### 2.1 Analysis of Method (ATHPM)

To introduce the ATHPM, we consider most general differential equation subject to condition

$$\begin{aligned} &Lu(x,t) + Ru(x,t) + Nu(x,t) = g(x,t) \\ &u(x,0) = \alpha, \quad u_t(x,0) = \beta, \end{aligned} \qquad (2)$$

From definition of Aboodh transform on Eq.(2), we have

$$A\{Du(x,t)\} = \frac{1}{v^2}A\{g(x,t)\} +$$
$$\frac{1}{v^2}A\{\alpha\} + \frac{1}{v^3}A\{\beta\} - \frac{1}{v^2}A\{Ru(x,t) + Nu(x,t)\}. \qquad (3)$$

Taking the inverse Aboodh transform on Eq. (3)

$$u(x,t) = F(x,t) - A^{-1}\left(\frac{1}{v^2}A\{Ru(x,t) + Nu(x,t)\}\right). \qquad (4)$$

Where *F(x,t)* symbolizes the sum of other terms.

Therefore, considering the unknown solution as

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t), \qquad (5)$$

and after decomposing the nonlinear term as

$$Nu(x,t) = \sum_{n=0}^{\infty} p^n H_n(u), \qquad (6)$$

Where *Hn(u)* are given by:

$$H_m(u_0,u_1,u_2,\dots,u_n) = \frac{1}{n!}\frac{\partial}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty}p^i u_i\right)\right]_{p=0}, \qquad (7)$$
$$n=0,1,2,\dots$$

Eq. (5) takes the form

$$p^n u_n(x,t) = G(x,t) - \qquad (8)$$
$$\sum_{n=0}^{\infty} p\left(A^{-1}\left(\frac{1}{v^2}A\left\{R\sum_{n=0}^{\infty}p^n u_n(x,t) + \sum_{n=0}^{\infty}p^n H_n(u)\right\}\right)\right).$$

Equivalently, we have

$$\begin{aligned} &p^0 : u_0(x,t) = G(x,t), \\ &p^1 : u_1(x,t) = A^{-1}\left(\frac{1}{v^2}A\{Ru_0(x,t) + H_0(u)\}\right), \\ &p^2 : u_2(x,t) = A^{-1}\left(\frac{1}{v^2}A\{Ru_1(x,t) + H_1(u)\}\right), \\ &p^3 : u_3(x,t) = A^{-1}\left(\frac{1}{v^2}A\{Ru_2(x,t) + H_2(u)\}\right), \end{aligned} \qquad (9)$$

Finally, the series solution is

$$u(x,t) = u_0(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) + \ldots \quad (10)$$

### 2.1. Numerical Examples

We consider some problems in order to presents the effectiveness and applicability of proposed Aboodh transform homotopy perturbation method.

**Example 4.1**

A response takes in a region in two reactors in series. The reactors are nicely mixed but are no longer at consistent state. The unsteady-state mass stability for each stirred tank reactor are given in the shape of system of differential equations are

$$\begin{aligned} Du(t) &= \frac{1}{\tau}(CA_0 - u) - \beta u \\ Dv(t) &= -\frac{1}{\tau} v - \beta u \\ Dw(t) &= \frac{1}{\tau}(u - w) - \beta w \\ Dy(t) &= \frac{1}{\tau}(v - y) - \beta y, \end{aligned} \quad (11)$$

Subject to constraints

*$CA_1(0)=u(0)CA_2(0)=w(0)=0, CB_1(0)=y(0)=0, CB_2(0)=y(0)=0$* (12)

Consider $CA_0$ is equal to 10, = 0.1and = 5.

By putting the values of $CA_0$ and we get,

$$\begin{aligned} Du(t) &= 2 - \frac{3}{10}u(t) \\ Dv(t) &= -\frac{1}{5}v(t) - \frac{1}{10}u(t) \\ Dw(t) &= \frac{1}{5}u(t) - \frac{3}{10}w(t) \\ Dy(t) &= \frac{1}{5}v(t) - \frac{3}{10}y(t) \end{aligned}$$

By applying the Aboodh and inverse Aboodh transformation we get,

$$u(t) = 2t - A^{-1}\left\{\frac{3}{10v}A\{u(t)\}\right\}$$

$$v(t) = -\frac{1}{10}t^2 + \frac{1}{100}t^3 - \frac{3}{4000}t^4 - A^{-1}\left\{\frac{1}{5v}A\{v(t)\}\right\}$$

$$w(t) = \frac{1}{5}t^2 - \frac{1}{50}t^3 + \frac{3}{20{,}000}t^4 - A^{-1}\left\{\frac{3}{10v}A\{w(t)\}\right\}$$

$$y(t) = -\frac{1}{50}t^3 + \frac{1}{1200}t^4 - \frac{19}{300{,}000}t^5 + \frac{17}{3{,}000{,}000}t^6 - \frac{1}{7{,}000{,}000}t^5 - A^{-1}\left\{\frac{3}{10v}A\{y(t)\}\right\} \quad 14)$$

Applying Homotopy Perturbation method and consequently, we get

$$\begin{aligned} p^0; u_0(x,t) &= 2t, \\ p^1; u_1(x,t) &= -\frac{3}{10}t^2, \\ p^2; u_2(x,t) &= \frac{3}{100}t^3, \end{aligned}$$

$$\begin{aligned} p^0; v_0(x,t) &= -\frac{1}{10}t^2 + \frac{1}{100}t^3 - \frac{3}{4000}t^4, \\ p^1; v_1(x,t) &= \frac{1}{150}t^3 - \frac{1}{2000}t^4 + \frac{3}{20,000}t^5, \\ p^2; v_2(x,t) &= -\frac{1}{3000}t^4 + \frac{1}{50,000}t^5 - \frac{1}{2,000,00}t^6, \end{aligned}$$

$$\begin{aligned} p^0; w_0(x,t) &= \frac{1}{5}t^2 - \frac{1}{50}t^3 + \frac{3}{20,000}t^4, \\ p^1; w_1(x,t) &= -\frac{1}{50}t^3 + \frac{3}{200}t^4 - \frac{3}{1,00,000}t^5, \\ p^2; w_2(x,t) &= \frac{3}{2000}t^4 - \frac{3}{10,000}t^5 + \frac{3}{6000000}t^6 \end{aligned}$$

$$\begin{aligned} p^0; y_0(x,t) &= \frac{1}{150}t^3 + \frac{1}{1200}t^4 - \frac{19}{300000}t^5 + \frac{17}{3000000}t^6 - \frac{1}{7000000}t, \\ p^1; y_1(x,t) &= 5\times10^{-4}t^4 - 5\times10^{-5}t^5 + 3.166666667\times10^{-6}t^6 - 2.428571429t^7 \\ &\quad + 5.357142857\times10^{-8}t^8 \end{aligned}$$

The series solution of (14) is

$$u(t) = 2t - \frac{3}{10}t^2 + \frac{3}{100}t^3 + \ldots$$

$$v(t) = -\frac{1}{10}t^2 + \frac{1}{60}t^3 - \frac{19}{12000}t^4 + \frac{17}{100{,}000}t^5 - \frac{1}{200{,}000}t^6 + \ldots$$

$$w(t) = \frac{1}{5}t^2 - \frac{1}{25}t^3 + \frac{333}{20{,}000}t^4 - \frac{33}{100{,}000}t^5 + \frac{1}{2{,}000{,}000}t^6 + \ldots$$

Y(t)= 6.666666667 $10^{-3}t^3$+1.33333333 $10^{-3}t^4$
1.1333334 $t$
$t$ $t$
57142857 $10^{8}t^8$+...

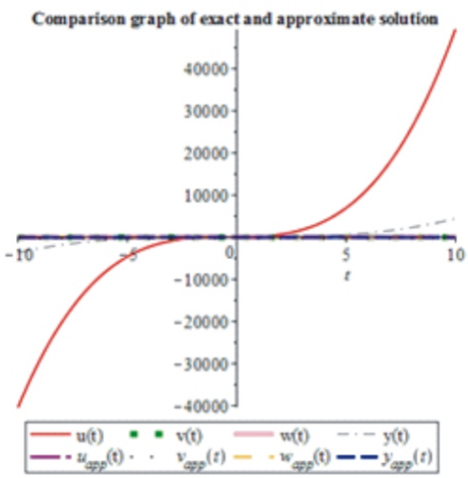


**Figure1:** *Shows the comparison of analytical approximate solution (15) of the system of first order unsteady-state mass stability differential equations of example 4.1 through the usage of Aboodh transform homotopy perturbation method (ATHPM) and exact solution*

**Example 4.2:**

The concentrations of three reactants are in the form of a gadget of nonlinear FDEs

$$Du(t) = - k_1u + k_2 vw$$
$$Dv(t) = - k_3u - k_4 vw - k_5v^2$$
$$Dw(t) = k_6v^2$$

where $k_1, k_2, k_3, k_4, k_5$ *and* $k_6$ are constant parameters ($k_1 = 0.04$, $k_2 = 0.01$, $k_3 = 400$, $k_4 = 100$, $k_5 = 30{,}000$ and $k_6 = 30$)

With given conditions

$$u(0) = 1,\ v(0) = 0,\ w(0)=0 \tag{17}$$

By putting initial conditions and the values of

$K_1, k_2, k_3, k_4, k_5$ *and* $k_6$

$$Du(t) = - 0.04\, u(t)$$
$$Du(t) = - 400\, u(t) - 30{,}000v^2(t)$$
$$Dw(t) = - 300v^2(t)$$

According to the above described procedure, we ge

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = 1 - p\left(A^{-1}\left\{\frac{1}{v}A\left\{0.04\sum_{n=0}^{\infty} p^n u_{n-1}(t)\right\}\right\}\right)$$
$$\sum_{n=0}^{\infty} p^n v_n(x,t) = 1 - 400t + 8t^2 - 0.106t^3 + 0.00106t^4 - p\left(A^{-1}\left\{\frac{30{,}000}{v}A\left\{\sum_{n=0}^{\infty} p^n H_n(t)\right\}\right\}\right)$$
$$\sum_{n=0}^{\infty} p^n w_n(x,t) = p\left(A^{-1}\left\{\frac{1}{v}A\left\{30\sum_{n=0}^{\infty} p^n H_n(t)\right\}\right\}\right).$$

$$H_0 = v_0^2,\ H_1 = 2v_0v_1,\ H_2 = v_1^2 + 2v_0v_2.$$

As a result

$$p^0; u_0(x,t) = 1,\ p^1; u_1(x,t) = -0.04t,\ p^2; u_2(x,t) = ($$
$$p^3; u_3(x,t) = 0.0000106t^3,$$
$$\vdots,$$
$$p^0; v_0(x,t) = 1 - 400t + 8t^2 - 0.106t^3 + 0.00106t^4,$$
$$p^1; v_1(x,t) = -30{,}000t + 12\times10^7 t^2 - 1.60016\times10^9 t^3 + 0.480015\times10^8 t^4 - 0.892812t^5,$$
$$+12720t^6 - 120.84t^7 + 0.8427t^8 - 0.003745t^9,$$
$$\vdots,$$
$$p^0; w_0(x,t) = 0,$$
$$p^1; w_1(x,t) = 30t + 88800t^2 + 8521600160t^3 - 535.2t^4,$$
$$p^2; w_2(x,t) = 1800000t^2 - 7.19964\times10^{10} t^4 + 1151961840t^5 - 127196682t^6 + 10902.5714t^7$$
$$\vdots,$$

The optained series solution is

$$u(t) = 1 - 0.04t + 0.0008t^2 - 0.0000106t^3 + \ldots$$
$$v(t) = 1 + 29600t - 11999992t^2 - 1.60016\times10^9 t^3 - 48001500t^4 + \ldots$$
$$w(t) = 30t + 1888800t^2 + 8521609160t^3 - 7.195644054\times10^{10} t^4 + 1151961840t^5 + \ldots$$

(20)

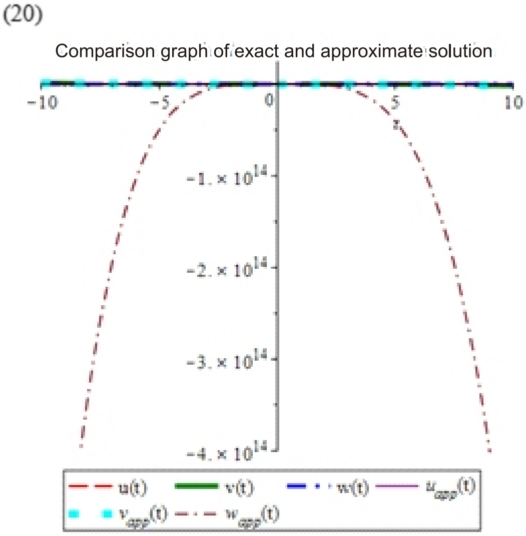


**Figure 2:** *Depicts the comparison of approximate solution (20) of first order of nonlinear system in example 4.3 by using Aboodh transforms homotopy*

**Example 4.4:**

Finally, the system of the nonlinear biochemical reaction model as

$$Du(t) = -u + (\beta - \alpha)v + uv$$
$$Dv(t) = \frac{1}{\gamma}(u - \beta v - uv), \quad (24)$$

with conditions

$$u(0) = 1, \;\; v(0) = 0 \quad (25)$$

Using $=0.375$, $=1$, $=0.1$, we get

$$Du(t) = -u + 0.625\,v + uv$$
$$Dv(t) = 10(u - v - uv) \quad (26)$$

According to the method Eq. (26) as

$$\sum_{n=0}^{\infty} p^n u_n(t) = 1 + p\left(A^{-1}\left\{\frac{1}{v}A\left\{\begin{matrix}-\sum_{n=0}^{\infty} p^n u_{n-1}(t) + \\ 0.625\sum_{n=0}^{\infty} p^n v_{n-1}(t) + \sum_{n=0}^{\infty} p^n H_n(t)\end{matrix}\right\}\right\}\right)$$

$$\sum_{n=0}^{\infty} p^n v_n(t) = p\left(A^{-1}\left\{\frac{1}{v}A\left\{10\left(\begin{matrix}\sum_{n=0}^{\infty} p^n u_{n-1}(t) - \\ \sum_{n=0}^{\infty} p^n v_{n-1}(t) - \sum_{n=0}^{\infty} p^n H_n(t)\end{matrix}\right)\right\}\right\}\right) \quad (27)$$

Where He's polynomials are

$$H_0 = u_0 v_0$$
$$H_1 = u_1 1(u_0 v_0 + u_0 v_0)$$

Eq. (27) gives the result as

$$p^0; u_0(t) = 1,$$
$$p^1; u_1(t) = 1 - 10t,$$
$$p^2; u_2(t) = 1 + 9t + 8.125t^2,$$
$$p^3; u_3(t) = 1 - t - 1.375t^2 - 7.08334t^3,$$
$$p^4; u_4(t) = 1 - t + 3.625t^2 + 5.4375t^3 + 0.88650t^4,$$

$$p^0; v_0(t) = 0,$$
$$p^1; v_1(t) = 10t,$$
$$p^2; v_2(t) = 10t - 55t^2,$$
$$p^3; v_3(t) = 10t + 50t^2 + 22.708334\,t^3,$$
$$p^4; v_4(t) = 10t - 10t^2 - 10.625\,t^3 - 3.8975225\,t^4,$$

The series solution will be

$$u(t) = 5 - 3t + 10.375\,t^2 - 1.64584\,t^3 + 0.88650t^4 + ..$$
$$v(t) = 40t - 15t^2 + 12.083334\,t^3 - 3.8975225\,t^3 + .. \quad (28)$$

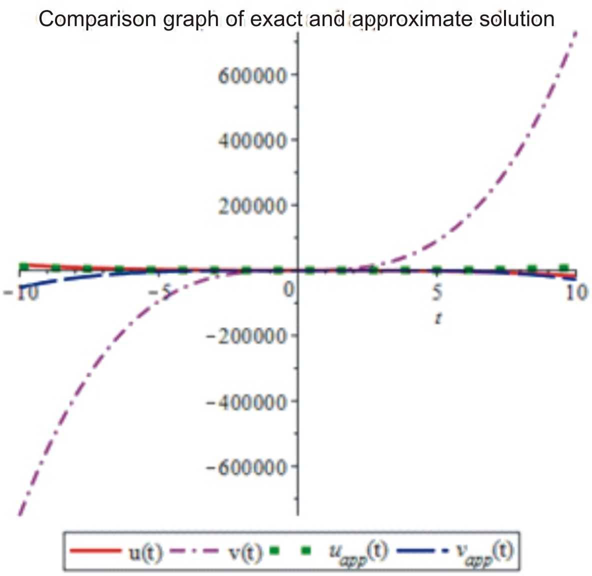


**Figure 4:** *Comparison of analytical approximate solution (28) of system of nonlinear differential equations of example 4.4 which representing a nonlinear reaction by using Aboodh transforms homotopy perturbation method (ATHPM) and exact solution*

**4. Conclusions:**

The primary purpose is to exhibit the applicability of the ATHPM method to assemble an analytical solution for some chemical engineering equations. The analytical expressions for the concentration of reactants were derived by using the Aboodh transform homotopy perturbation method. The coupling of homotopy perturbation technique and the Aboodh transform, proved very fine to handle analytically nonlinear chemical engineering equations. From the presented graphical comparison, considering the first few terms only of the approximate solution we concluded that with the addition of more terms in the approximate solution will improve the accuracy of the result. The fast convergence of the proposed method suggests that the approach is trustworthy. We analyze that this proposed technique is nicely acceptable for such physical problems as it presents solution in less number of iterations. From the assimilated result, it may be concluded that the applications of suggested method may be extended to twodimension and three-dimension geometrics and coupled processes of first-order and higher order chemical reactions and to other nonlinear mathematical issues springing up of other fields also.

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